

A Prediction from the Type III See-saw Mechanism

S.M. Barr*

Bartol Research Institute

University of Delaware

Newark, DE 19716

Ilja Dorsner†

The Abdus Salam International Centre for Theoretical Physics

Strada Costiera 11, 34014 Trieste, Italy

A simple ansatz that is well-motivated by group-theoretical considerations is proposed in the context of the type III neutrino see-saw mechanism. It results in predictions for m_s/m_b and m_τ/m_b that relates these quantities to the masses and mixings of neutrinos.

Simple unified models based on $SO(10)$ and related groups can lead to the so-called “type III see-saw mechanism” for neutrino masses [1]. In the most general case the type III mechanism leads to a light neutrino mass matrix given by the formula $M_\nu = -(M_N H + H^T M_N^T)(u/\Omega)$, where M_N is the Dirac mass matrix of the neutrinos, H is a dimensionless complex three-by-three matrix and u/Ω is the ratio of a weak-scale vacuum expectation value to a GUT-scale vacuum expectation value (VEV). In a subsequent paper the type III see-saw mechanism was shown to have certain advantages for leptogenesis, in particular allowing resonant enhancement without fine-tuning the form of neutrino mass matrices [2]. In the simplest case, where a minimal set of Higgs fields breaks $B - L$, one has $H = I$ and the type III see-saw formula takes the simple form

$$M_\nu = -(M_N + M_N^T) \frac{u}{\Omega}. \quad (1)$$

The main problem in constructing predictive models of neutrino masses and mixings with the usual “type I” see-saw formula [3], $M_\nu = -M_N M_R^{-1} M_N^T$, is to relate the Majorana mass matrix of the right-handed neutrinos M_R , with its six complex parameters, to measurable quantities. There are very special models, such as the recently much studied “minimal $SO(10)$ models”, where there is such a relationship [4]. (For an exhaustive list of references on “minimal $SO(10)$ models” see [5].) And the study of leptogenesis may tell us something about the structure of M_R (although leptogenesis has only a single data point to work with). In general, however, the lack of information

*Electronic address: smbarr@bxclu.bartol.udel.edu

†Electronic address: idorsner@ictp.trieste.it

about M_R is a problem for the predictivity of type I see-saw models. (The so-called “type II see-saw mechanism” [6] assumes the existence of $SU(2)_L$ -triplet Higgs fields with small VEVs that couple directly to $\nu_L \nu_L$. About the type II mechanism we have nothing to say in this paper.)

What makes the simplest version of the type III formula, given in Eq. (1), so remarkable and appealing is that it does not involve the masses of the superheavy right-handed neutrinos at all. As a consequence, the simplest type III formula opens the possibility of constructing models of quark and lepton masses that are extremely predictive. In particular, in models based on $SO(10)$ or other groups that unify an entire family within a single multiplet, the Dirac mass matrix of the neutrinos M_N is typically closely related by the grand-unification symmetries to the mass matrices (also of Dirac type, of course) of the up quarks, down quarks and charged leptons, which we will denote respectively as M_U , M_D , and M_L . It is therefore possible in many models (for examples, see [7, 8, 9]) to predict the matrix M_N from a knowledge of the masses and mixings of the quarks and the masses of the charged leptons. This would allow, if Eq. (1) holds, the complete prediction of the mass ratios and mixing angles of the neutrinos with no free parameters.

In this paper we will not be so ambitious. We have not found so far a full three-family model that is as predictive as that and where all the predictions (or “postdictions”) are consistent with experiment. Rather, as an illustration of the possibilities of the type III framework, we will present here a simple ansatz for the heavier two families that is well motivated by group-theoretical considerations. This ansatz leads to two interesting predictions that are consistent with present experimental data. Before presenting the ansatz, we very briefly review the type III see-saw mechanism and formula.

In models based on $SO(10)$, there are two ways that the right-handed neutrinos N_i^c ($i = 1, 2, 3$) can get mass, either through a renormalizable term such as $\mathbf{16}_i \mathbf{16}_j \overline{\mathbf{126}}_H$, or through a higher-dimension effective operator such as $\mathbf{16}_i \mathbf{16}_j \overline{\mathbf{16}}_H \overline{\mathbf{16}}_H / M_{GUT}$. The former allows automatic conservation of “matter parity”, whereas the latter makes do with smaller multiplets of Higgs fields. In the latter case, the effective $d = 5$ operator arises most simply from integrating out three or more $SO(10)$ -singlets, which we will denote by $\mathbf{1}_a$ or S_a , that have the couplings $F_{ia} \mathbf{16}_i \mathbf{1}_a \overline{\mathbf{16}}_H$ and $(M_S)_{ab} \mathbf{1}_a \mathbf{1}_b$. If only the Standard-Model-singlet component of the $\overline{\mathbf{16}}_H$ has a non-zero VEV, and we denote it by $\Omega \sim M_{GUT}$, then one has the familiar “double see-saw” mass matrix:

$$\mathcal{L}_{neutrino} = (\nu_i, N_i^c, S_a) \begin{pmatrix} 0 & (M_N)_{ij} & 0 \\ (M_N^T)_{ij} & 0 & F_{ib}\Omega \\ 0 & F_{aj}\Omega & (M_S)_{ab} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_b \end{pmatrix}. \quad (2)$$

By integrating out the superheavy fields N_i^c and S_a , one obtains $M_\nu = -M_N M_R^{-1} M_N^T$, where $M_R = -(F\Omega)M_S^{-1}(F\Omega)^T$. This is just the type I see-saw formula, with an effective M_R .

Now, if we assume that the $SU(2)_L$ -doublet Higgs field contained in $\overline{\mathbf{16}}_H$ also gets a non-zero VEV (and there is no fundamental reason why it should not), and we denote it by u , then the double see-saw mass matrix takes the form:

$$\mathcal{L}_{neutrino} = (\nu_i, N_i^c, S_a) \begin{pmatrix} 0 & (M_N)_{ij} & F_{ib}u \\ (M_N^T)_{ij} & 0 & F_{ib}\Omega \\ F_{aj}^T u & F_{aj}^T \Omega & (M_S)_{ab} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_b \end{pmatrix}. \quad (3)$$

In this case, it is easy to show that the effective mass matrix of the light neutrinos takes the form:

$$M_\nu = -M_N M_R^{-1} M_N^T - (M_N + M_N^T) \frac{u}{\Omega}, \quad (4)$$

where, as before, $M_R = -(F\Omega)M_S^{-1}(F\Omega)^T$. The first term is the usual type I see-saw contribution, and the second term is the type III see-saw contribution. (The origin of the type III term can be simply understood as follows. One can eliminate the νS and $S\nu$ entries in Eq. (3), i.e. the entries Fu and $F^T u$, by doing a rotation of the (ν_i, N_i^c) basis by an angle $\theta \cong \tan \theta = u/\Omega$. That reduces the matrix in Eq. (3) to the same form as Eq. (2), but with the zeros replaced by terms of the type III form.) Both the type I and the type III terms in Eq. (3) are formally of order M_W^2/M_{GUT} . However, since the elements of M_N are actually small compared to M_W because of small Yukawa couplings (except perhaps for the third family), and since M_N comes in quadratically in the type I term but only linearly in the type III term, one might expect the type III term to dominate for generic values of the parameters. Moreover, in the limit that the elements of M_S are small compared to the GUT scale, the type I contribution becomes small. As was pointed out in [2], that is a good limit for the purposes of enhancing leptogenesis. It is therefore plausible that one can neglect the type I term, and we shall do so.

Now let us turn to the ansatz for the various Dirac mass matrices. Suppose that these have the form (neglecting the small masses of the first family)

$$M_U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a \\ 0 & b & 1 \end{pmatrix} m_U, \quad M_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & c \\ 0 & d & 1 \end{pmatrix} m_D, \quad (5)$$

$$M_N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g \\ 0 & h & 1 \end{pmatrix} m_U, \quad M_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & e \\ 0 & f & 1 \end{pmatrix} m_D, \quad (6)$$

where the “texture zero” in the 22 elements can be enforced by an abelian family symmetry, either discrete or continuous. We will say more on this later. And further suppose that the entries satisfy the conditions

$$a + b = g + h, \quad c + d = e + f. \quad (7)$$

The relations given in Eq. (7) are not arbitrary, but follow from group-theory if the elements of the mass matrices come from no operators except of the following simple types:

- (1) $\mathbf{16}_i \mathbf{16}_j \mathbf{10}_H$,
- (2) $\mathbf{16}_i \mathbf{16}_j \mathbf{120}_H$,
- (3) $\mathbf{16}_i \mathbf{16}_j \mathbf{10}_H \mathbf{45}_H / M_{GUT}$.
- (4) $\mathbf{16}_i \mathbf{16}_j \mathbf{16}'_H \mathbf{16}_H / M_{GUT}$,

Eq. (7) is satisfied no matter how many operators there are of any of these types. Any operator of type (1) gives $a = g$, $b = h$, $c = e$, and $d = f$, thus satisfying Eq. (7). Any operator of type (2) gives contributions that are flavor-antisymmetric (since the $\mathbf{120}$ is in the antisymmetric product of two spinors). Consequently, it gives $a + b = 0$, $c + d = 0$, $e + f = 0$, and $g + h = 0$, thus also satisfying Eq. (7) in a trivial way.

Any operator of type (3) gives contributions of the form $f_i f_j^c v_f [\alpha Q(f) + \beta Q(f^c)]$. Here Q is that generator of $SO(10)$ to which the VEV of the adjoint Higgs field ($\mathbf{45}_H$) is proportional; $Q(f)$ is the value of this charge for the fermion f ($= u, d, \ell^-, \nu$); $v_f = v_u$ or v_d depending on whether f is of the weak-isospin up or down type; and the coefficients α and β depend on the way the $SO(10)$ indices are contracted in the operator. Thus an operator of type (3) will give, for instance, $c + d \propto (\alpha + \beta)(Q(d) + Q(d^c))v_d$ and $e + f \propto (\alpha + \beta)(Q(\ell^-) + Q(\ell^+))v_d$. Since the terms $d_i d_j^c H_d$ and $\ell^- \ell^+ H_d$ must be invariant under the charge Q , it follows that $Q(d) + Q(d^c) = -Q(H_d) = Q(\ell^-) + Q(\ell^+)$, and so $c + d = e + f$, satisfying Eq. (7). In the same way it is easily seen that $a + b = g + h$.

Finally, consider an operator of type (4). One of the spinor Higgs fields (say the unprimed one) gets a superlarge VEV that breaks $SO(10)$ down to $SU(5)$. The effective operator that results is then of the form $(\alpha \mathbf{10}_i \bar{\mathbf{5}}_j + \beta \bar{\mathbf{5}}_i \mathbf{10}_j) \bar{\mathbf{5}}_H$, where the coefficients depend on the contraction of $SO(10)$ indices in the original operator. This gives no contribution to a , b , g , and h , and gives contributions to the other parameters of the form $c = f$ and $d = e$ (note the transposition between M_D and M_L). Again, such contributions satisfy Eq. (7).

Simple low-dimension operators that could give contributions *not* satisfying Eq. (7) are $\mathbf{16}_i \mathbf{16}_j \bar{\mathbf{126}}_H$ (if the $SU(5)$ $\bar{\mathbf{45}}$ contained in the $\bar{\mathbf{126}}_H$ got a non-zero VEV), and $\mathbf{16}_i \mathbf{16}_j \bar{\mathbf{16}}'_H \bar{\mathbf{16}}_H / M_{GUT}$.

One might ask why we do not include the effects of operators of even higher dimension, such as $\mathbf{16}_i \mathbf{16}_j \mathbf{10}_H \mathbf{45}_H^n / M_{GUT}^n$, which are not obviously smaller than the dimension-five operators that we included in our analysis, and which would not satisfy Eq. (7) in general. Such operators ought indeed to be present. However there are reasons that one might expect them to be small, as we now explain. Consider the operator $\mathbf{16}_2 \mathbf{16}_3 \mathbf{10}_H \mathbf{45}_H / M_{GUT}$, which will contribute to the 23 and 32 elements in our illustrative model. Since these elements are somewhat small compared to the 33 elements, either the effective Yukawa couplings in this term are small or the ratio $\langle \mathbf{45}_H \rangle / M_{GUT}$ must be, or both. This operator can arise from integrating out a pair of multiplets $\mathbf{16}' + \overline{\mathbf{16}}'$ that have GUT-scale mass, as follows. Suppose the terms $a \mathbf{16}_3 \mathbf{16}' \mathbf{10}_H$, $b \mathbf{16}_2 \overline{\mathbf{16}}' \mathbf{45}_H$, and $M \mathbf{16}' \overline{\mathbf{16}}'$. Integrating out the primed fields gives an effective operator $ab \mathbf{16}_2 \mathbf{16}_3 \mathbf{10}_H \mathbf{45}_H M^{-1} [1 + |b \mathbf{45}_H / M|^2]^{-1/2}$. If b or $\langle \mathbf{45}_H \rangle / M$ are small, then the higher order operators are highly suppressed. This is not to say that operators of higher dimension must always be unimportant, but it is a plausible assumption easily implemented that they can be neglected.

To return to the texture zero in the 22 elements, it could be enforced, for example, by a $U(1)$ family symmetry under which the $\mathbf{16}_3$, $\mathbf{10}_H$ and $\mathbf{16}'_H$ are neutral; the $\mathbf{16}_2$ has charge +1; and the $\mathbf{45}_H$, $\mathbf{16}_H$, and $\mathbf{120}_H$ have charge -1.

Given the simplest type III form (Eq. (1)), and the ansatz of Eqs. (5), (6), and (7), one has

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (a+b) \\ 0 & (a+b) & 2 \end{pmatrix} \frac{um_U}{\Omega}. \quad (8)$$

$$M_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & e \\ 0 & (c+d-e) & 1 \end{pmatrix} m_D, \quad (9)$$

with the quark matrices M_U and M_D given by Eq. (5). Consequently, to the extent that we can ignore the first family, the five parameters a , b , c , d , and e determine the following mass ratios and mixings of the second and third families: m_c/m_t , m_s/m_b , V_{cb} , m_μ/m_τ , m_2/m_3 (the neutrino mass ratio), $U_{\mu 3} \equiv \sin \theta_{atm}$, and m_τ/m_b . Therefore there are two predictions. (We assume all the parameters are real.)

What we have done is use the values of the five quantities m_c/m_t , m_μ/m_τ , V_{cb} , m_2/m_3 , and θ_{atm} to solve for the five parameters a , b , c , d , and e . Then we have used the resulting values of those parameters to “predict” the values of m_s/m_b and m_τ/m_b at the GUT scale. For the first

three inputs (m_c/m_t , m_μ/m_τ , and V_{cb}), which are fairly well known, we have taken the central experimental values and run them up to the GUT scale, assuming low energy supersymmetry. The running depends significantly on the value of $\tan\beta$, and so we make a predictions for a particular set of values of $\tan\beta$ that span the interesting range: 2, 3, 10, 25, 40, and 57. The other two inputs (m_2/m_3 and θ_{atm}) come from neutrino oscillation experiments (see the reviews [10, 11]) and have rather large error bars. (For example, $\theta_{atm} = 45^\circ \pm 6^\circ$ at M_Z .) We have assumed hierarchical spectrum for neutrino masses with $m_1 < 0.007$ eV. Under this assumption the RGE evolved values of m_2/m_3 and θ_{atm} at the GUT scale remain within 3% of their low-scale values even for large $\tan\beta$. (For relevant renormalization group equations see [12, 13].) Hence, we drop their running and allow these two inputs to vary within the experimentally allowed range and plot our predictions for m_s/m_b and m_τ/m_b as a function of them in Fig. 1.

We take the experimental values of the quarks from Ref. [14], except for m_s for which we use the results of lattice calculations as given in Ref. [15] and double the error as suggested in Ref. [16]. The values of the CKM angles and the charged lepton masses are taken from PDG 2004 [17]. In presenting our results for m_s/m_b and m_τ/m_b in Fig. 1, we give the percentage by which the predicted GUT values differ from the RGE-evolved experimental central values.

In doing the renormalization group running we assume that all the sparticles have mass of 1 TeV. From M_Z to 1 TeV, the running is done at one loop, assuming the Standard Model with two Higgs doublets. From 1 TeV to the GUT scale (taken to be 2×10^{16} GeV) we do a two-loop running assuming the MSSM. The gauge coupling constants are taken from PDG 2004 [17]. We present one example of the RGE evolution in Table I.

It should be noted that, even with the assumption that we are making that the parameters a , b , c , d , and e are real, there are discrete ambiguities of the relative signs of these parameters. (The overall sign does not matter.) The choice that gives by far the best fits is $(a, b, c, d, e) = \pm(-, +, +, -, -)$. A typical set of values is $a \simeq -0.00455$, $b \simeq +0.9$, $c \simeq +0.04$, $d \simeq -0.45$, $e \simeq -0.55$, and $f \equiv c + d - e \simeq +0.14$.

Note that the value of a is very small. It is this that accounts for the smallness of m_c/m_t . One way that a might be small naturally (i.e. without fine-tuning) using only the set of operators that satisfy Eq. (7) is by means of an operator of the form $\mathbf{16}_2 \mathbf{16}_3 \mathbf{10}_H \mathbf{45}_H / M_{GUT}$, where $\langle \mathbf{45}_H \rangle \propto Q = I_{3R} + \epsilon(B - L)$, $\epsilon \ll 1$, where I_{3R} and $B - L$ are the familiar $SO(10)$ generators (I_{3R} the diagonal generator of $SU(2)_R$ in the Pati-Salam subgroup, and $B - L$ the baryon minus lepton number), and where the fields are contracted in such a way that this generator Q acts on the field $\mathbf{16}_2$. (This would happen, for instance if the effective operator came from integrating out a $\mathbf{16}' + \overline{\mathbf{16}}'$ having

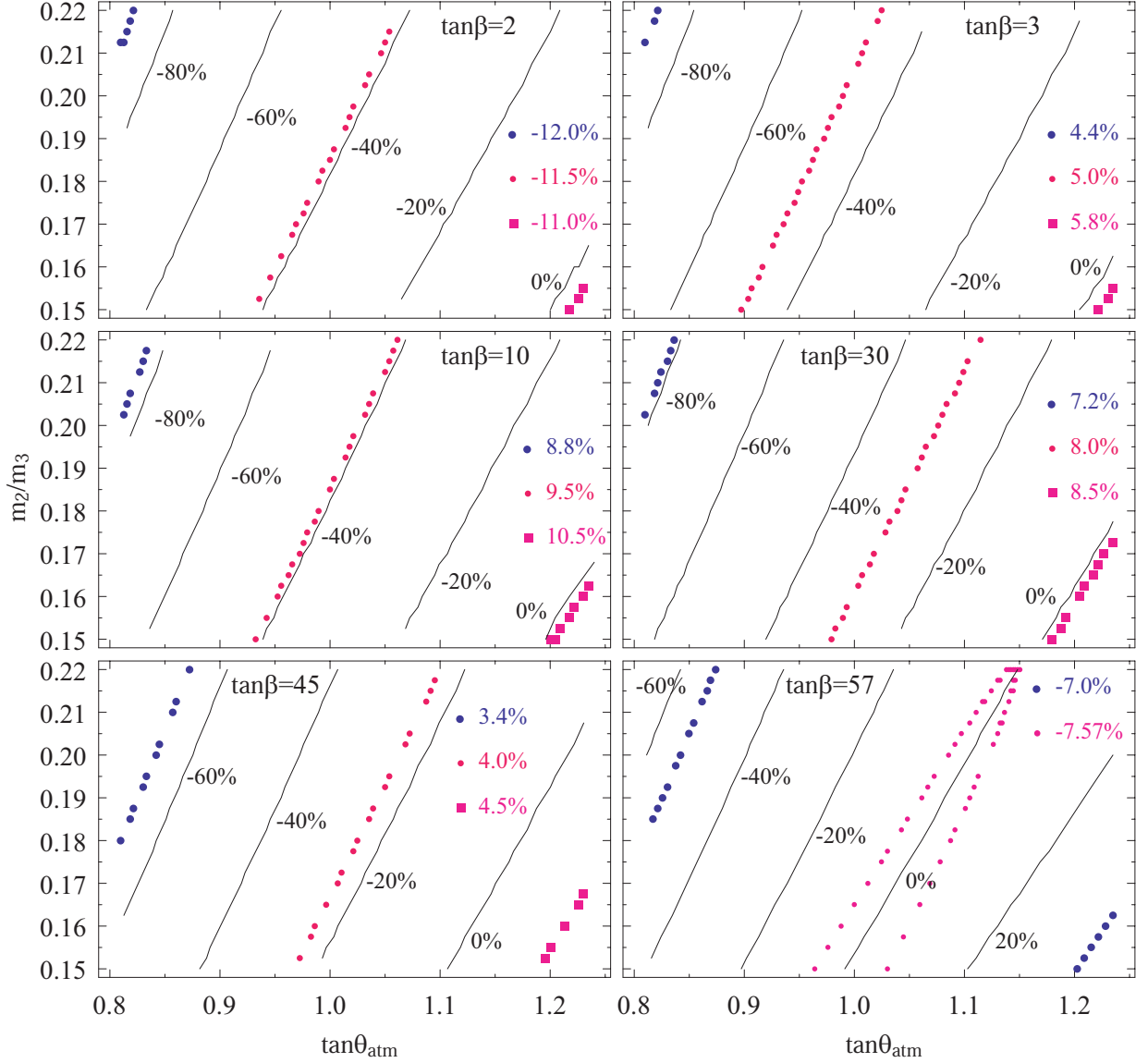


FIG. 1: Plots of constant values of m_s/m_b (solid lines) and m_τ/m_b (dots) in the $\tan\theta_{atm}-m_2/m_3$ plane for $\tan\beta = 2, 3, 10, 30, 45$, and 57 . The percents represent the departure of the fitted ratio from the central value obtained from the RGE evolution. In the case of the m_τ/m_b ratio the uncertainty in the b and τ mass allows this departure to be within $\pm 8\%$ range.

the couplings $\mathbf{16}_2\overline{\mathbf{16}}'\mathbf{45}_H$, $\mathbf{16}_3\mathbf{16}'\mathbf{10}_H$, and $M\mathbf{16}'\overline{\mathbf{16}}'$.) This operator would give off-diagonal mass terms for the up quarks proportional to $Q(u)u_3^c u_2 + Q(u^c)u_2^c u_3$. Since, $I_{3R}(u) = 0$, this would give $a/b = O(\epsilon)$.

The values of m_s/m_b that we predict are satisfyingly close to the experimental (lattice) results. A couple of things should be noted in this regard. First, it was long thought that the Georgi-

TABLE I: Input values at the M_Z scale vs. the GUT scale values ($M_{GUT} = 2 \times 10^{16}$ GeV) for $\tan \beta = 45$. In the fermion case we use indicated errors at the M_Z scale to extract corresponding errors for individual fermions at the GUT scale. No correlation is taken into account.

	$\mu = M_Z$	$\mu = M_{GUT}$
$\tan \beta(\mu)$	45.00	35.36
$v_u(\mu)$ (GeV)	174.05	117.33
$v_d(\mu)$ (GeV)	3.87	3.32
$m_u(\mu)$ (MeV)	$2.33^{+0.42}_{-0.45}$	$0.73^{+0.13}_{-0.14}$
$m_c(\mu)$ (MeV)	677^{+56}_{-61}	212^{+18}_{-19}
$m_t(\mu)$ (GeV)	181^{+13}_{-13}	93^{+42}_{-17}
$m_d(\mu)$ (MeV)	$4.69^{+0.60}_{-0.66}$	$1.52^{+0.19}_{-0.21}$
$m_s(\mu)$ (MeV)	$53.8^{+13.3}_{-13.3}$	$17.4^{+4.3}_{-4.3}$
$m_b(\mu)$ (GeV)	$3.00^{+0.11}_{-0.11}$	$1.34^{+0.08}_{-0.07}$
$m_e(\mu)$ (MeV)	$0.48684727^{+0.00000014}_{-0.00000014}$	$0.35620421^{+0.00000010}_{-0.00000010}$
$m_\mu(\mu)$ (MeV)	$102.75140^{+0.00033}_{-0.00033}$	$75.20781^{+0.00024}_{-0.00024}$
$m_\tau(\mu)$ (GeV)	$1.74669^{+0.00030}_{-0.00027}$	$1.45111^{+0.00032}_{-0.00029}$
$ (V_{CKM})_{12}(\mu) $	0.2200	0.2199
$ (V_{CKM})_{13}(\mu) $	0.00367	0.00300
$ (V_{CKM})_{23}(\mu) $	0.0413	0.0337

Jarlskog [18] relation $(m_s/m_b)_{GUT} = \frac{1}{3}(m_\mu/m_\tau)_{GUT}$ gave a good fit to the data in SUSY GUT models. However, the recent lattice calculations have given results for m_s that are typically only about 0.6 times the typical values that had been obtained by previous methods. Because of that, many models which were constructed in the past to give the Georgi-Jarlskog result, would be off from the current central experimental/lattice results for m_s/m_b by about +60%. That compares to the values we are getting, which agree with the current central value of m_s/m_b for some of the allowed $(m_2/m_3) - (\theta_{atm})$ parameter space, and are within 20% for a large part of that space.

A second point is that inclusion of the first family is likely to push up the predicted value of m_s/m_b by about 5%. The reason is that empirically the relation for the Cabbibo angle $\theta_C \simeq \sqrt{m_d/m_s}$ is known to work very well [19]. As is well-known, this formula arises naturally if the 11 element of the down quark mass matrix vanishes and the 12 and 21 elements are approximately equal [20]. But then diagonalizing the 12 block of the down quark mass matrix will push up the value of the 22 element by a factor of $(1 + |m_d/m_s|)$.

In any event, we see that further improvement in the measurement of the θ_{atm} , δm_{atm}^2 , δm_{sol}^2 ,

and the lattice results for m_s , together with an eventual determination of $\tan\beta$ will allow our simple ansatz, given in Eq. (7), to be tested.

-
- [1] E. Akhmedov, M. Lindner, E. Schnapka, and J.W.F. Valle, *Phys. Rev.* **D53**, 2752 (1996); *Phys. Lett.* **B368**, 270 (1996); S. M. Barr, *Phys. Rev. Lett.* **92**, 101601 (2004); M. Malinský, J.C. Romão and J.W.F. Valle, hep-ph/0506296.
 - [2] C. H. Albright and S. M. Barr, *Phys. Rev.* **D69**, 073010 (2004).
 - [3] P. Minkowski, *Phys. Lett.* **B67**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, ed. P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p.315; T. Yanagida, in *Proc. Workshop on Unified Theory and the Baryon Number of the Universe*, Tsukuba, Japan, 1979, ed. O. Sawada and A. Sugramoto (KEK Report No. 79-18, Tsukuba, 1979), p.95; R. N. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980); S. L. Glashow, in *Quarks and Leptons*, Cargèse (July 9-29, 1979), ed. M. Levy et al. (Plenum, New York, 1980), p.707.
 - [4] K. S. Babu and R. N. Mohapatra, *Phys. Rev. Lett.* **70**, 2845 (1993); B. Bajc, G. Senjanovic, and F. Vissani, *Phys. Rev. Lett.* **90**, 051802 (2003); C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, *Phys. Lett. B* **588**, 196 (2004); S. Bertolini, M. Frigerio and M. Malinsky, *Phys. Rev. D* **70**, 095002 (2004); K. S. Babu and C. Macesanu, hep-ph/0505200.
 - [5] C. S. Aulakh, arXiv:hep-ph/0506290.
 - [6] J. Schechter and J. W. F. Valle, *Phys. Rev.* **D22**, 2227 (1980); G. Lazarides, Q. Shafi, and C. Wetterich, *Nucl. Phys.* **B181**, 287 (1981); R. N. Mohapatra and G. Senjanovic, *Phys. Rev.* **D23**, 165 (1981).
 - [7] C. H. Albright and S. M. Barr, *Phys. Rev.* **D58**, 013002 (1998); C. H. Albright, K. S. Babu, and S. M. Barr, *Phys. Rev. Lett.* **81**, 1167 (1998); C. H. Albright and S. M. Barr, *Phys. Lett.* **B452**, 287 (1999).
 - [8] K. S. Babu, J. C. Pati, and F. Wilczek, *Nucl. Phys.* **B566**, 33 (2000).
 - [9] G. Anderson, S. Raby, S. Dimopoulos, L. J. Hall, and G. D. Starkman, *Phys. Rev.* **bf D49**, 3660 (1994).
 - [10] A. Strumia and F. Vissani, arXiv:hep-ph/0503246.
 - [11] G. L. Fogli, E. Lisi, A. Marrone and A. Palazzo, arXiv:hep-ph/0506083.
 - [12] K. S. Babu, C. N. Leung and J. T. Pantaleone, *Phys. Lett. B* **319**, 191 (1993) [arXiv:hep-ph/9309223].
 - [13] S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Nucl. Phys. B* **674**, 401 (2003) [arXiv:hep-ph/0305273].
 - [14] H. Fusaoka and Y. Koide, *Phys. Rev.* **D57**, 3986 (1998).
 - [15] R. Gupta and K. Maltman, *Int. J. Mod. Phys.* **A16S1B**, 591 (2001).
 - [16] H. D. Kim, S. Raby and L. Schradin, *Phys. Rev. D* **69**, 092002 (2004).
 - [17] S. Eidelman *et al.* [Particle Data Group], *Phys. Lett. B* **592** (2004) 1.
 - [18] H. Georgi and C. Jarlskog, *Phys. Lett.* **B86**, 297 (1979).

- [19] R. Gatto, G. Sartori, and M. Tonin, *Phys. Lett.* **B28**, 128 (1968); R. J. Oakes, *ibid.* **B29**, 683 (1969).
- [20] S. Weinberg, *Trans. N.Y. Acad. Sci. Ser. II* **38**, 185 (1977); F. Wilczek and A. Zee, *Phys. Lett.* **B70**, 418 (1977); H. Fritzsch, *ibid.* **B70**, 436 (1977).